

Systematic study of the elliptic flow parameter using a heavy-ion collision model

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Elliptic flow parameter, v_2 is considered as a sensitive probe for early dynamics of the heavy-ion collision. In this work we have discussed the effect of detector efficiency, procedure of centrality determination, effect of resonance decay, procedure to obtain event plane resolution on the measured v_2 by standard event plane method within the framework of a transport model. The measured value of v_2 depends on the detector efficiency in particle number counting. The effect of centrality determination is found to be negligible. The method of event-by-event correction of event plane resolution for wide centrality bin yields in results closer to the true value of v_2 . The effect of resonance decay is seen to decrease the v_2 of π , K and p . We also propose a procedure to correct for an event bias effect on v_2 while comparing the minimum bias centrality v_2 values for different multi-strange hadrons. Finally we have presented a model based confirmation of the recently proposed relation between v_2 obtained using event plane method and scalar product method to the true value of v_2 .

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I. INTRODUCTION

The study of the azimuthal angle distribution for hadrons produced in high energy heavy-ion collisions is considered as a very useful observable for understanding the properties of the hot and dense matter formed in the collisions [1–8]. The second Fourier coefficient of the azimuthal angle (ϕ) distribution of the particles produced in heavy-ion collision with respect to the reaction plane angle (Ψ) is called as the elliptic flow parameter (v_2) [9]. It is defined as

$$v_2 = \langle \langle \cos(2(\phi - \Psi)) \rangle \rangle = \langle \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle \rangle, \quad (1)$$

where p_x and p_y are the x and y components of the particle momenta, respectively. The $\langle \langle \rangle \rangle$ denotes the average over all particles in all events. The Ψ is an estimate of the angle subtended by the plane formed by impact parameter and beam (z) axis with respect to the x -axis. The magnitude of v_2 is found to be sensitive to the equation of state, thermalization, transport coefficients of the medium, and initial conditions in the heavy-ion collisions [10–12].

Since v_2 is an important observable in high energy heavy-ion collisions, it is necessary to study the effect of various experimental conditions towards the measurement of its true value. In this work we study the effect of finite particle number counting (detector efficiency), method of centrality determination, effect of correcting for finite resolution in obtaining Ψ and effect of resonances. For this study we use transport based models, which provides an ideal set up to study and quantify the above effects. From the models we know the true value of v_2 at the same time it provides information of particles produced in heavy-ion collisions that can be treated in a manner similar to the actual experimental conditions. For minimum bias collisions the event class (in terms of average initial spatial anisotropy) having a rare heavy

particle like Ω baryon could be different from that having a proton (copious produced). This may necessitate an appropriate correction to the measured v_2 of different hadrons for wide centrality class (0-80%) so that they can be compared among themselves. In addition, we also use this model framework to verify the recently proposed relation that v_2 obtained using event plane method approaches the root-mean square v_2 (as obtained from the scalar product method) in the small event plane resolution limit while it approaches the mean v_2 value in the high event plane resolution limit [13].

The paper is organised as follows. In section II, we describe the transport based models used in this study. The effect of detector efficiency on measured v_2 is presented in section III. In section IV, we investigate the effect of centrality selection procedure on measured v_2 . The event plane resolution correction methods, event bias correction method and effect of resonance decay on v_2 has been discussed in sections V, VI and VII respectively. In section VIII we discuss the recently proposed relation between v_2 obtained from event plane and scalar product method. Finally section IX summarises our findings.

II. MODEL DESCRIPTION

The A Multi Phase Transport (AMPT) model [14] (version: 25t7d) uses the same initial conditions as in HIJING [15]. However the minijet partons are made to undergo scattering before they are allowed to fragment into hadrons. The string melting (SM) version of the AMPT model is based on the idea that for energy densities beyond a critical value of ~ 1 GeV/fm³, it is difficult to visualize the coexistence of strings (or hadrons) and partons. Hence the need to melt the strings to partons. This is done by converting the mesons to a quark and anti-quark pair, baryons to three quarks etc. The scattering of the quarks are based on parton cascade [14]. Once the interactions stop, the partons then hadronizes

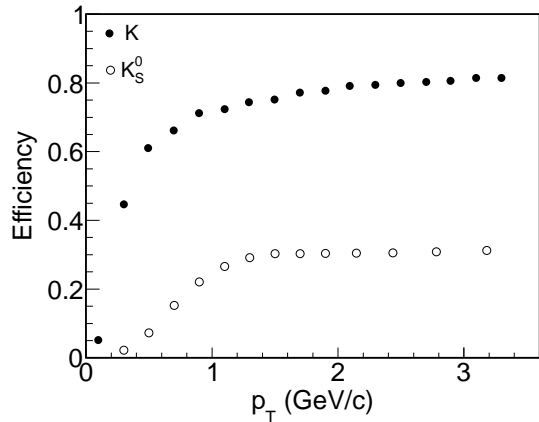


FIG. 1: Typical particle reconstruction efficiency as function of p_T for charged kaon and K_S^0 at midrapidity in Au+Au collisions at high energy.

through the mechanism of parton coalescence. The interactions between the minijet partons in AMPT model and those between partons in the AMPT-SM model could give rise to substantial v_2 . All results presented here using this model is for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at midrapidity ($-0.5 < \eta < 0.5$) with total event statistics of 1.8 million minimum bias (0-80%) events. The true event plane angle is fixed to zero degree for the simulation results presented.

The Ultra relativistic Quantum Molecular Dynamics (UrQMD) model (version: 3.3) is based on a microscopic transport theory where the phase space description of the reactions are important [16]. It allows for the propagation of all hadrons on classical trajectories in combination with stochastic binary scattering, color string formation and resonance decay. It incorporates baryon-baryon, meson-baryon and meson-meson interactions, the collisional term includes more than 50 baryon species and 45 meson species. This model is used to understand the resonance decay effect on measured v_2 . The analysis makes use of 1.5 million minimum bias (0-80%) events for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at midrapidity.

III. EFFECT OF DETECTOR EFFICIENCY

In this section we discuss the effect of detector efficiency on measured v_2 in a typical heavy-ion experiment. The AMPT model is used to simulate Au+Au collision and introduced the effect of finite detector efficiency on particle number counting. A realistic detector efficiency for reconstruction of charged kaon and K_S^0 as a function of the transverse momentum (p_T) of the measured hadron from a typical heavy-ion experiment [17, 18] as shown in Fig. 1 is considered. The reconstruction efficiency for

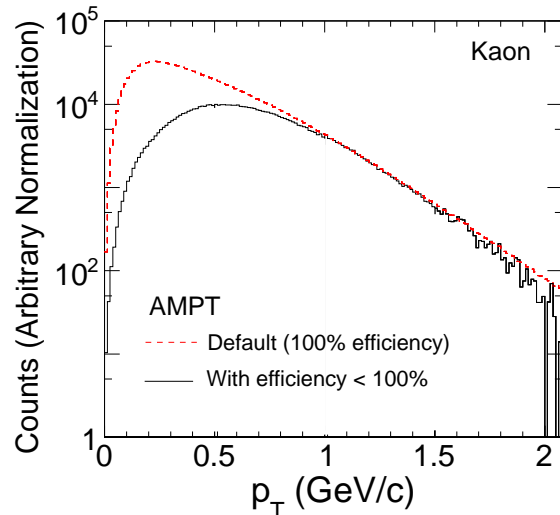


FIG. 2: (Color online) Kaon yield as function of p_T from AMPT model for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Red dash line corresponds to the yield of kaon obtained directly from the AMPT model and black solid line corresponds to the kaon yield after incorporating the finite K_S^0 reconstruction efficiency values as shown in Fig. 1. Yields of the two spectra are normalised at $p_T = 1$ GeV/c.

neutral kaons, which have to be reconstructed from the decayed charged pion daughters are smaller compared to those from more directly reconstructed charged kaons. Two sets of events were considered, one with 100% reconstruction efficiency versus p_T termed as “default” and the other where the kaon tracks in a given event for a given p_T range were randomly removed as per the efficiency shown in Fig. 1. The resultant yield of the charged kaons from AMPT model with K_S^0 reconstruction efficiency effect incorporated has been compared to the default case in Fig. 2. The two distributions have been normalised at their respective yield values at $p_T = 1$ GeV/c. The effect of finite particle reconstruction efficiency can be seen from the shape of yield vs. p_T distribution below $p_T = 1$ GeV/c and as the efficiency values are constant with p_T beyond 1 GeV/c the spectra shape are similar at high p_T (consistent with Fig. 1).

The elliptic flow of charged kaon has been calculated for three different condition: (a) with 100% particle reconstruction efficiency (labeled as default), (b) with charged kaon reconstruction efficiency and (c) with K_S^0 reconstruction efficiency. Figure 3 shows the comparison of kaon v_2 for the above three different cases. The kaon v_2 from AMPT for default case, with charged kaon reconstruction efficiency and with K_S^0 reconstruction efficiency are shown as solid black circle, open blue square and open red circle, respectively. The bottom panel of Fig. 3 shows the ratios of default kaon v_2 to that obtained with two different particle reconstruction efficiency. There is a change due to the finite particle reconstruction efficiency

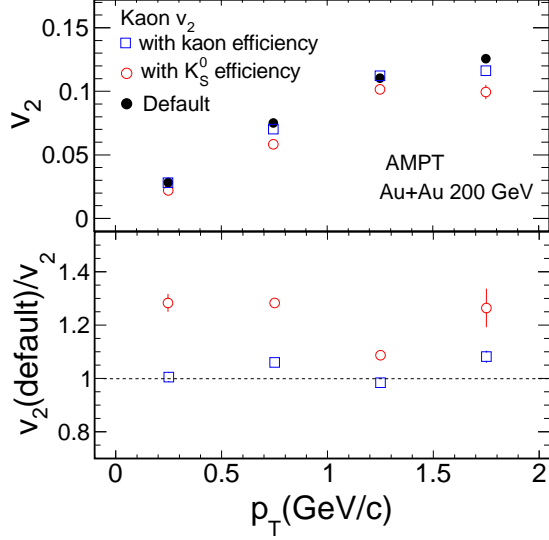


FIG. 3: (Color online) v_2 of kaon as function of p_T in Au+Au collision for 0-80% from AMPT model in three different condition: 100% kaon reconstruction efficiency (default), with charged kaon reconstruction efficiency and with K_S^0 reconstruction efficiency.

on the measured v_2 . The change in v_2 due to K_S^0 reconstruction efficiency is about 10% to 30% while for the results using the higher charged kaon reconstruction efficiency the change is less than 5%. Our study shows a need for correcting the effect of difference in finite reconstruction efficiency between charged kaon and K_S^0 , before the measured v_2 values are compared in the experiments.

IV. EFFECT OF CENTRALITY DETERMINATION PROCEDURE

Determination of centrality selection in experiments is found to play an important role in measurements related to particle correlations [19]. Specifically when the same particles that are used for the correlation studies also forms a subset of the particles used to obtain the centrality selection. Hence it is important to study the effect of centrality determination procedure on measurement of v_2 . Further different experiments use different methods of centrality selection. Hence such a study is necessary to see if measured v_2 values across different experiments can be compared.

The simulated charged particle average v_2 ($\langle v_2 \rangle$) values are obtained for three different ways for centrality selection. They are: (a) centrality obtained using charged particle multiplicity within $|\eta| < 0.5$ (labeled as centrality 1), centrality obtained using charged particle multiplicity within $|\eta| > 0.5$ and $|\eta| < 1.0$ (labeled as centrality 2), and that obtained using spectator neutrons (labeled as

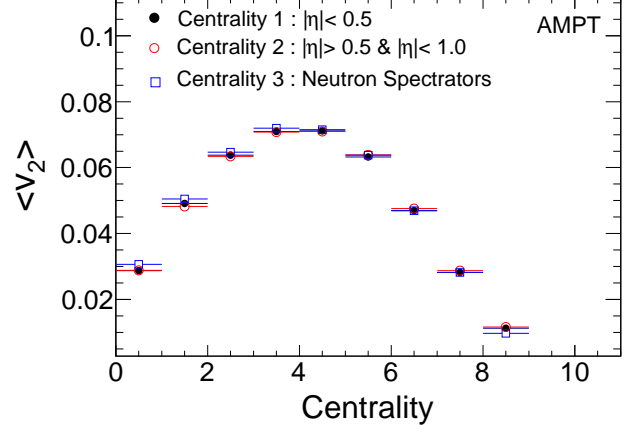


FIG. 4: (Color online) $\langle v_2 \rangle$ of charged particles as function of collision centrality at midrapidity from AMPT model. The x-axis value of zero corresponds to most peripheral collisions (70-80%) and 9 corresponds to most central (0-5%) Au + Au collisions studied.

centrality 3). Experiments at the Relativistic Heavy-Ion Collider facility commonly uses these methods to select on collision centrality. Figure 4 show $\langle v_2 \rangle$ of charged particles measured at midrapidity ($|\eta| < 0.5$) as function of centrality for the above three different cases using AMPT model. We observed that the maximum difference in $\langle v_2 \rangle$ due to different centrality selection procedure is $\sim 2\%$. The agreement between the $\langle v_2 \rangle$ result from centrality 1 and centrality 2 shows there is no auto-correlation effect, due to using common set of particles for both the centrality determination and $\langle v_2 \rangle$ calculation.

V. EVENT PLANE RESOLUTION CORRECTION

Event plane (Ψ) is an estimation of true reaction plane (Ψ_r). As the estimated reaction plane fluctuates owing to finite number of particles, one has to correct observed v_2^{obs} by the corresponding event plane resolution. The event plane resolution is defined by the correlation of the event plane with the reaction plane [20]:

$$R = \langle \cos(2(\Psi - \Psi_r)) \rangle. \quad (2)$$

The reaction plane is not measurable in experiments hence resolution can not be calculated using above relation. To estimate the event-plane resolution we measure the correlation between the azimuthal angles of two sub-set groups of tracks, called sub-events (labeled as A and B):

$$R = \langle \cos(2(\Psi - \Psi_r)) \rangle = C \cdot \langle \cos(2(\Psi^A - \Psi^B)) \rangle. \quad (3)$$

where C is factor that depends on the multiplicity of the event, $\Psi^{A,B}$ are sub event plane angles and $\langle \rangle$ denote

the average over events. The resolution corrected v_2 is given as,

$$v_2 = \frac{\langle \cos(2(\phi - \Psi)) \rangle}{\langle \cos(2(\Psi^A - \Psi^B)) \rangle} = \frac{v_2^{obs}}{R}. \quad (4)$$

Most commonly used method for resolution correction for an average v_2 over a wider centrality range (like 0-80%) is

$$\langle v_2 \rangle = \frac{\langle v_2^{obs} \rangle}{\langle R \rangle}. \quad (5)$$

Here $\langle R \rangle$ is the mean resolution in that wide centrality bin and can be calculated as

$$\langle R \rangle = \frac{\sum N_i \langle R \rangle_i}{\sum N_i}. \quad (6)$$

where N_i and $\langle R \rangle_i$ is the multiplicity and resolution of the i^{th} narrow centrality bin (typical centrality bin widths of 5% or 10%). However as shown using AMPT model simulations for charged particles in Fig. 5, such a procedure does not recover back the true v_2 denoted as $v_2(RP)$.

Therefor another approach for event plane resolution correction for wide centrality bin has been proposed [21]. In this method resolution correction for wide centrality bin is done by dividing the term $\cos(2(\phi - \Psi))$ by the event plane resolution (R) for the corresponding centrality for each event.

$$\langle v_2 \rangle = \langle \frac{v_2^{obs}}{R} \rangle. \quad (7)$$

These two method would not yield the same value of $\langle v_2 \rangle$ because

$$\frac{\langle v_2^{obs} \rangle}{\langle R \rangle} \neq \langle \frac{v_2^{obs}}{R} \rangle. \quad (8)$$

Figure 5 shows charged particles v_2 as function of p_T for 0-80% centrality bin in Au + Au collisions. The red marker corresponds to v_2 measured with respect to true reaction plane. Open black and solid blue circle corresponds to v_2 measured with respect to event plane and resolution correction done using method described in equation 5 and 7, respectively. The v_2 measured with respect to true reaction plane is the actual v_2 in the AMPT model. The results in Fig. 5 shows that event-by-event resolution correction method using the equation 7 gives v_2 values closer to the true v_2 .

VI. EVENT BIAS CORRECTION

In the particular case of comparing the v_2 values for heavier hadrons such as Ω to those copiously produced such as pions or protons in minimum bias collisions there is an inherent bias towards the event class. This is illustrated in the top panel of Fig. 6. It shows the number

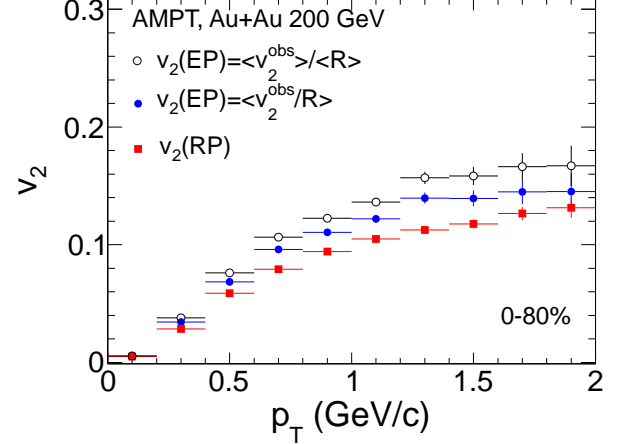


FIG. 5: (Color online) The v_2 of charged particles as function of p_T for 0-80% centrality in Au + Au collisions from AMPT model. The results for two different methods of event plane resolution correction are compared to the true v_2 values obtained using the known reaction plane angle in the model.

of events as function of particle multiplicity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from AMPT Model. The black, red, blue and green histogram corresponds to the events which contain at least one proton, ϕ , Ξ and Ω , respectively. The minimum bias multiplicity distribution with heavier hadrons like Ξ and Ω are very different from those for protons. The participant eccentricity (ε_{part}) obtained using the initial position of the participating nucleons [22, 23] is correlated with the particle multiplicity as seen from Fig. 6 bottom panel. Hence the average ε_{part} of events containing multi-strange hadron like Ω in 0-80% wide centrality would be smaller than the average ε_{part} determined for events containing protons. Since v_2 is driven by the anisotropy of the initial spatial geometry, therefore the event bias is naturally introduced when comparison is made between v_2 measured in a wide centrality bin especially for the rarely produced particles like Ω to that for protons. This event bias effect needs to be corrected before comparisons of minimum bias v_2 values for different types of hadrons. This bias could be corrected by normalising the measured v_2 by the ratio of standard average ε_{part} (for charged particle) to the average ε_{part} of the events which contains the particle of interest weighted by the corresponding yield. The correction factor in the present calculations using AMPT data for Ω is 1.15 where as for Ξ and ϕ it is 1.023 and 1.010, respectively. The lighter hadrons do not have a large event bias correction, due to their copious production in nuclear collisions at RHIC energy.

The physical consequence of such an event bias is shown in Fig. 7. The number of constituent quark scaling between Ω and proton which is naturally expected in AMPT model holds better at the intermediate p_T only

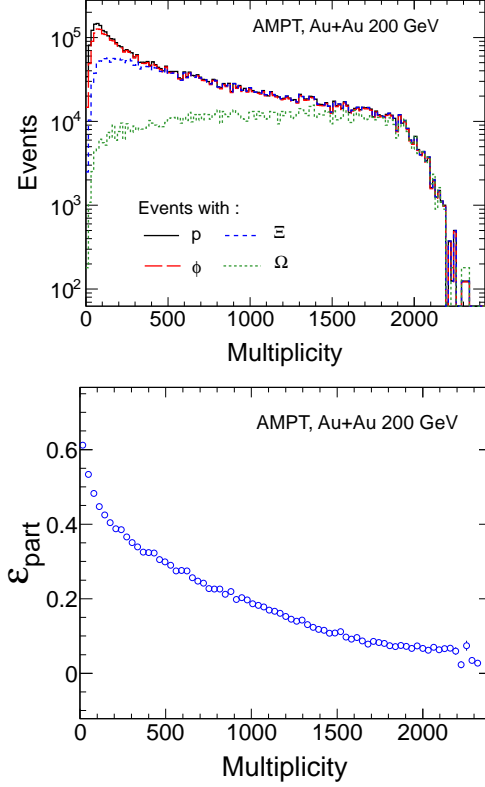


FIG. 6: (Color online) Top panel: Number of events as a function of particle multiplicity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from AMPT model. Bottom panel: Participant eccentricity as function of particle multiplicity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from AMPT model.

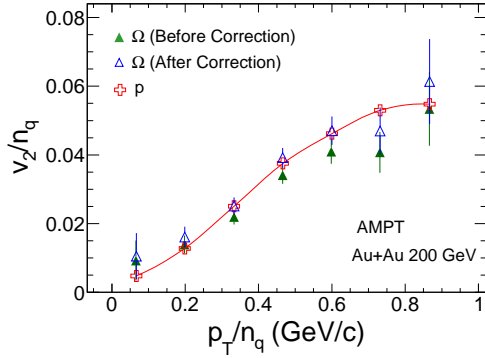


FIG. 7: (Color online) v_2/n_q as a function of p_T/n_q in 0-80% minimum bias Au+Au collisions ($|\eta| < 0.5$) at $\sqrt{s_{NN}} = 200$ GeV from AMPT model. n_q is the number of constituent quarks, equal to 3 here for the baryons.

after the event bias correction as described above.

VII. RESONANCE DECAY EFFECT

In the heavy-ion collision a large fraction of stable hadrons are from resonance decays. To study the effect of resonance decays on the elliptic flow of stable hadrons, we have used the UrQMD model.

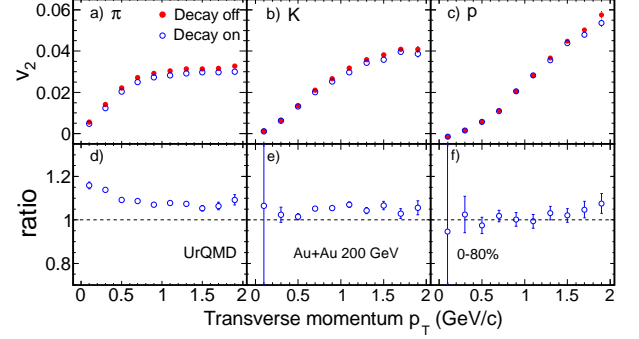


FIG. 8: (Color online) The v_2 of π , K and p as function of p_T at 0-80% centrality with decay off and decay on condition from minimum bias Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV from UrQMD model.

Specifically we study the effect of resonances, such as ρ , Λ , ϕ , η , Ω , Σ and Δ on measured v_2 of inclusive pion, kaon and proton. In the UrQMD model there is a option for switching off and on the decay of each resonance. Figure 8 shows v_2 of π , K and p as function of p_T with decay off and decay on condition in Au+Au collision from UrQMD model. The ratios shown in the lower panels of Fig 8, shows that there is change of 10% to 15% in the v_2 values of pion, less than 5% for the kaon and the v_2 values for the proton is almost unaffected. There is a decrease in v_2 values due to the decay of resonances. However, one could expect a higher value of v_2 from decay of resonances. The decay particle at given transverse momentum arises mostly from a resonance at higher momentum. The v_2 value in general increases with p_T . However, it seems the decay process being isotropic in the rest frame of the resonance, reduces the v_2 [24]. To understand the results better, we have further studied the effect from the decay of $\rho \rightarrow \pi^+ + \pi^-$ on v_2 of pions. This decay process is isotropic in the rest frame of the resonance and hence one can expect reduction in the momentum anisotropy of the daughter pions.

Figure 9 shows the v_2 of pion as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from UrQMD model for three different cases: (a) All resonances are decayed (shown by red inverted triangles), (b) ρ , Λ , η , Σ and Δ are not decayed (shown by solid blue circle) and, (c) Λ , η , Σ and Δ are not decayed (shown by black triangle). There is decrease in the v_2 values of pion due to the decay of $\rho \rightarrow \pi^+ + \pi^-$ as expected from the decay kinematics (comparison between cases (b) and (c)). However, the pion v_2 values increases due to decay of Λ , η , Σ and Δ (comparing between cases (a) and (c)).

Similarly we have observed that the decreases in kaons v_2 is due to $\phi \rightarrow K^+ + K^-$ decay (not shown here). There is an overall decrease in the measured value of pion and kaon v_2 due to the decay of the resonances.

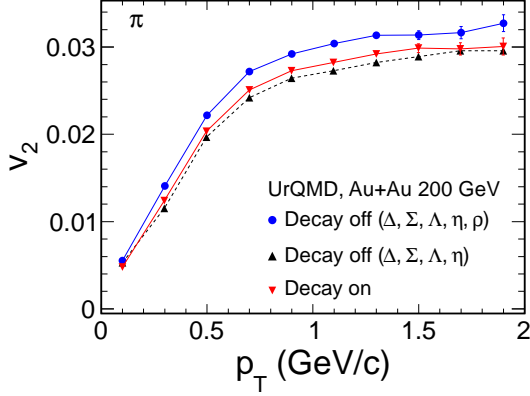


FIG. 9: (Color online) v_2 of pion as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for 0-80% centrality from UrQMD model. The different results shown corresponds to different contributions from resonance decay.

VIII. ELLIPTIC FLOW FROM SCALAR PRODUCT METHOD

In a scalar product method [25] of determining v_2 each event is partitioned into two sub events, labeled here as A and B. If Q_n^A and Q_n^B are the flow vectors of sub events A and B for the n^{th} harmonic then the correlation between the two sub events is given as

$$\langle Q_n^A Q_n^{B*} \rangle = \langle v_n^2 M^A M^B \rangle, \quad (9)$$

where M^A and M^B are the multiplicities for sub events A and B, respectively. Elliptic flow parameter in this method is given as

$$v_2(SP) = \frac{\langle Q_2 u_{2*} \rangle}{\sqrt{\langle Q_2^A Q_2^{B*} \rangle}}. \quad (10)$$

Here $Q_2 = \sum u_2^i$ and u_2^i is a unit vector associated with the i^{th} particle. The scalar-product method always yields the root-mean-square v_2 , regardless of the details of the analysis [13].

$$v_2(SP) = \frac{\langle Q_2 u_{2*} \rangle}{\sqrt{\langle Q_2^A Q_2^{B*} \rangle}} = \sqrt{\langle v_2^2 \rangle}. \quad (11)$$

But this is not true for $v_2(EP)$ measured by conventional event plane method. Recently it has been argued [13] that in the limit of perfect resolution (i.e. $R \rightarrow 1$)

$$v_2(EP) \rightarrow \langle v_2 \rangle, \quad (12)$$

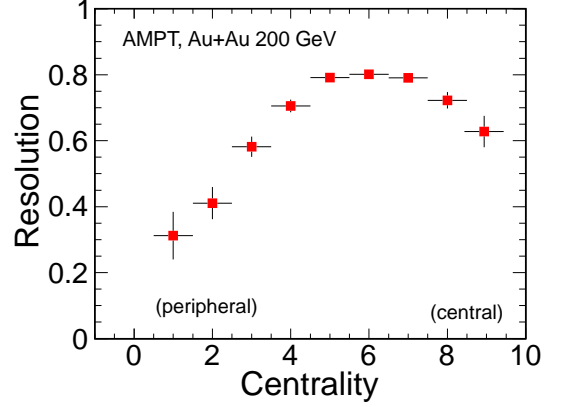


FIG. 10: (Color online) Second order event plane resolution as a function of centrality in Au+Au collisions at 200 GeV from AMPT model.

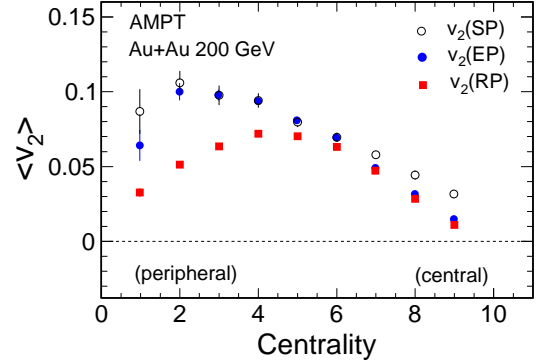


FIG. 11: (Color online) The elliptic flow of charged particle obtained using different methods as a function of centrality in Au+Au collisions at 200 GeV from AMPT model.

and in the limit of low resolution

$$v_2(EP) \rightarrow \sqrt{\langle v_2^2 \rangle}. \quad (13)$$

We have investigate this using AMPT model where the actual $\langle v_2 \rangle$ is known ($v_2(RP)$). The event plane resolution from AMPT model is shown in Fig. 10 for nine centrality bins, corresponding to 0-5%, 5-10%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%, 60-70% and 70-80% for the cross section. Resolution is poor for the peripheral centrality and maximum at mid-central and then slightly decreases for the most central collisions. This is due to the interplay of multiplicity and v_2 for different centrality bins. Figure 11 shows charged particle v_2 as a function of centrality in Au+Au collisions at 200 GeV from AMPT model using scalar product, event plane and reaction plane methods. The most central collisions studied corresponds to a value of 9 in the x -axis and most peripheral collisions corresponds to a value of 0 in the x -axis. For the peripheral collision where resolution is

poor, $v_2(EP)$ and $v_2(SP)$ are very close to each other that means $v_2(EP)$ is equivalent to $\sqrt{\langle v_2^2 \rangle}$. However for central to mid-central collisions where resolution is high, $v_2(EP)$ is closer to $v_2(RP)$ or $\langle v_2 \rangle$. The results are consistent with equations 12-13 and as proposed in Ref. [13].

IX. SUMMARY

We have presented a transport model based study of various effects on experimentally measured v_2 . The results are presented using Au+Au collisions at midrapidity at $\sqrt{s_{NN}} = 200$ GeV from AMPT and UrQMD model. We find that finite particle counting efficiency of the detectors used in real experiments affects the measured v_2 values. Specifically due to the difference in the reconstruction efficiencies of charged kaon and neutral kaons, the measured v_2 values could differ by 10-30% as a function of p_T . Experiments need to account for this inefficiencies while obtaining v_2 using event plane method, before comparing the results for measured hadrons with very different reconstruction efficiencies. We observe that the measured v_2 values remain insensitive to the method of centrality determination used in the experiments. However the procedure to correct for event plane resolution in wide centrality bin results affects the extracted v_2 values. Event-by-event resolution correction

seems to give v_2 values that are closer to the true v_2 values. The overall effect of resonance decay is to reduce the v_2 values relative to the true v_2 . This is dominated due to kinematic effect of the decay process being isotropic in the rest frame of the resonance and such resonances contributing more in terms of the yields of the measured hadrons. The minimum bias event class in terms of average initial ε_{part} value for events having rare heavier particle like Ω is different from those having protons. In order to appropriately compare the minimum bias v_2 values of various hadrons, we propose an event bias correction procedure. We also demonstrate that this procedure seems to work by showing that number of constituent quark scaling for Ω and protons v_2 . Finally we have demonstrated through the model study that in the limit of high resolution for event plane determination the $v_2(EP) \rightarrow \langle v_2 \rangle$ and in the limit of small event plane resolution the $v_2(EP) \rightarrow \sqrt{\langle v_2^2 \rangle}$.

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